

# A Petri Net View of Mobility

(FORTE 2005)

Charles Lakos

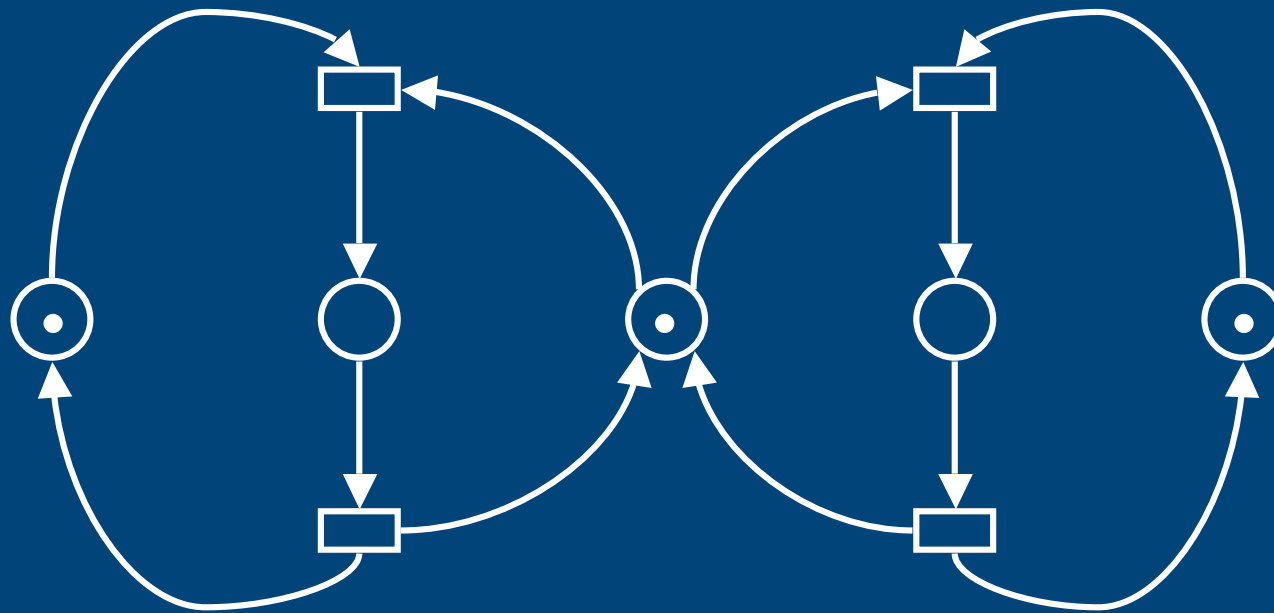
# Roadmap



- Introduction/reminder of Petri Net formalism
- Key issues of mobility
- Previous approach – the Hamburg group
  
- The proposal – based on modular nets
- Coloured version and the notion of garbage
  
- Conclusions

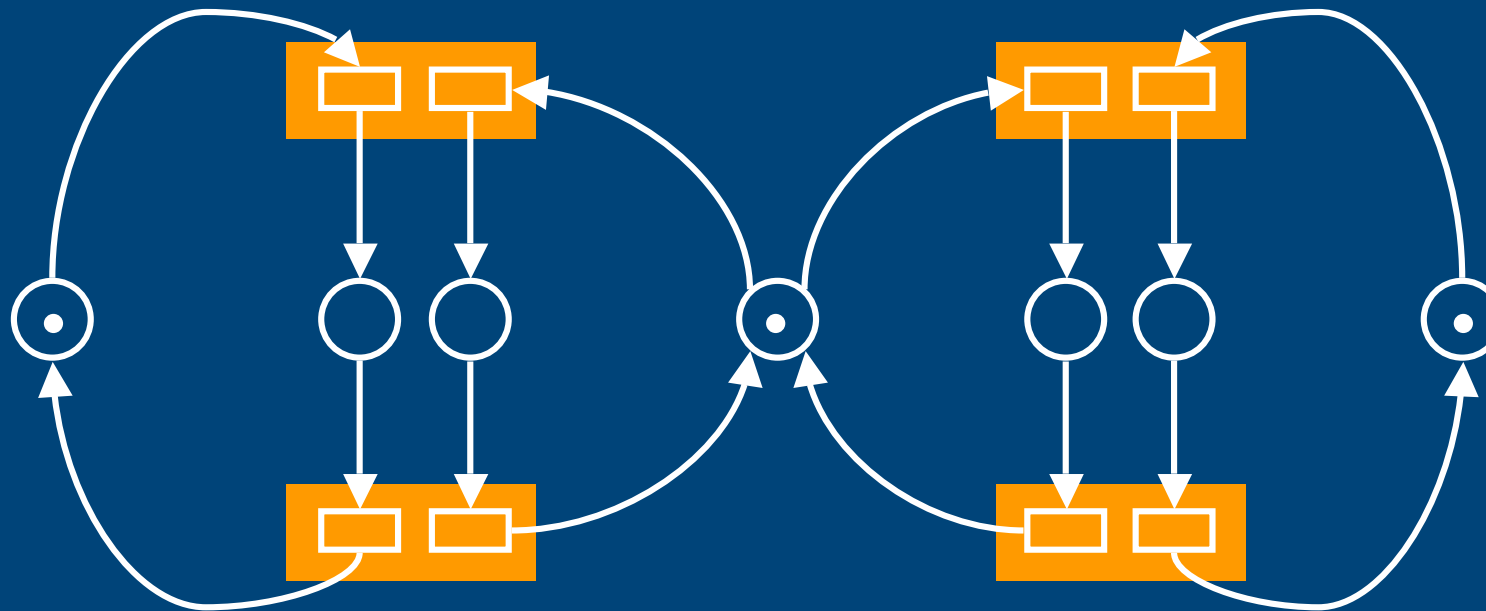
# Petri Nets

- Net structure – places, transitions, arcs
- System behaviour – markings, steps



# Petri Nets

- Modular structure – place fusion, transition fusion



# Mobile systems



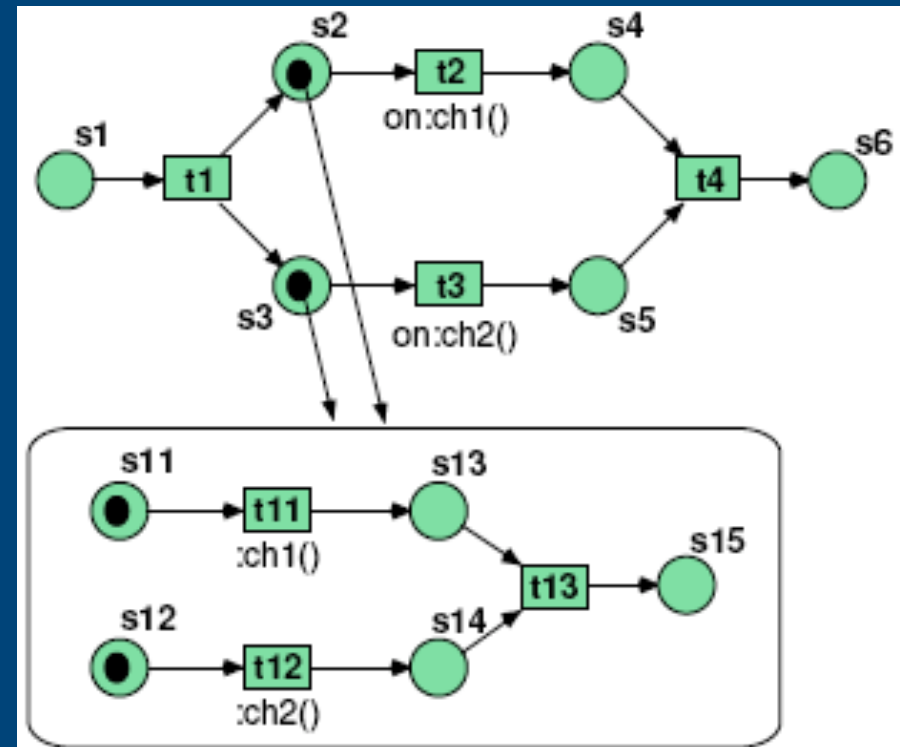
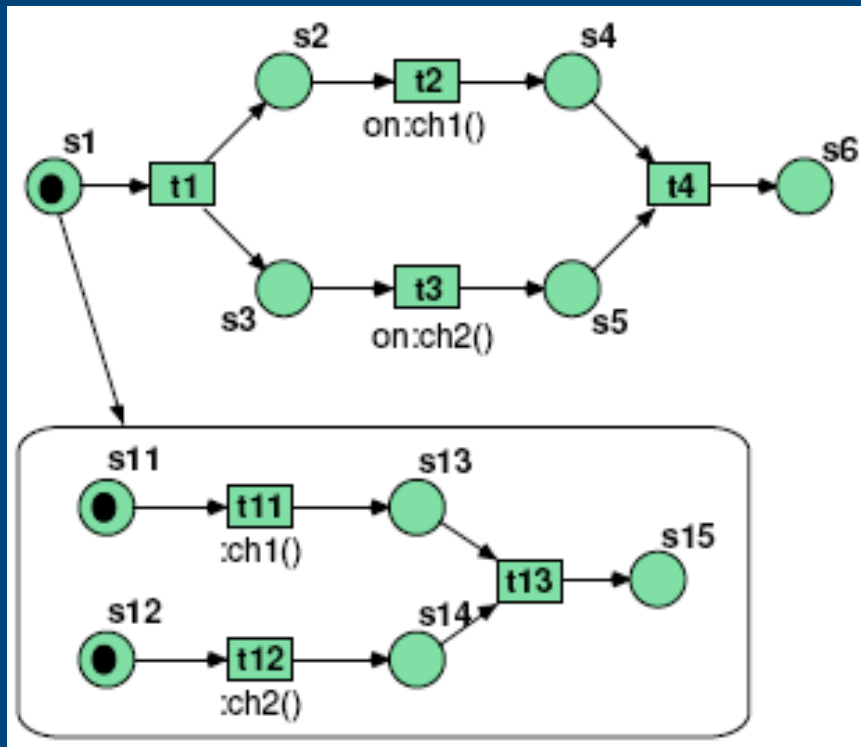
- Expose the interplay between *locality* and *connectivity* (Milner)
- **Connectivity** involves having a reference and being able to dereference it
- **Locality** constrains what you can dereference
- A simple and general Petri Net solution has proved elusive

# Nets-within-nets paradigm (Hamburg)

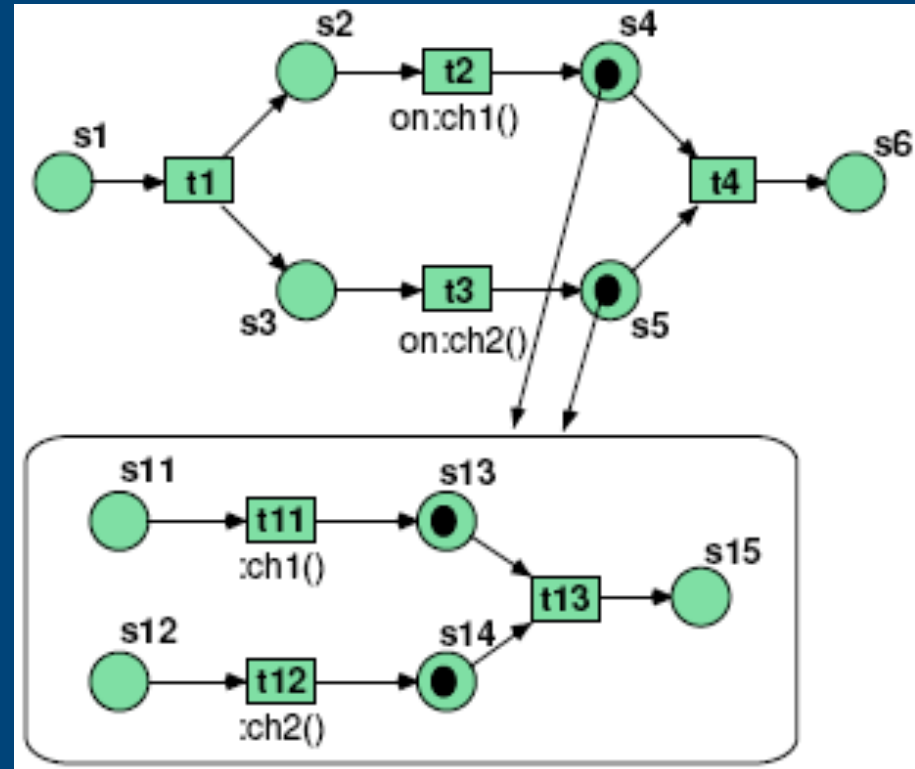
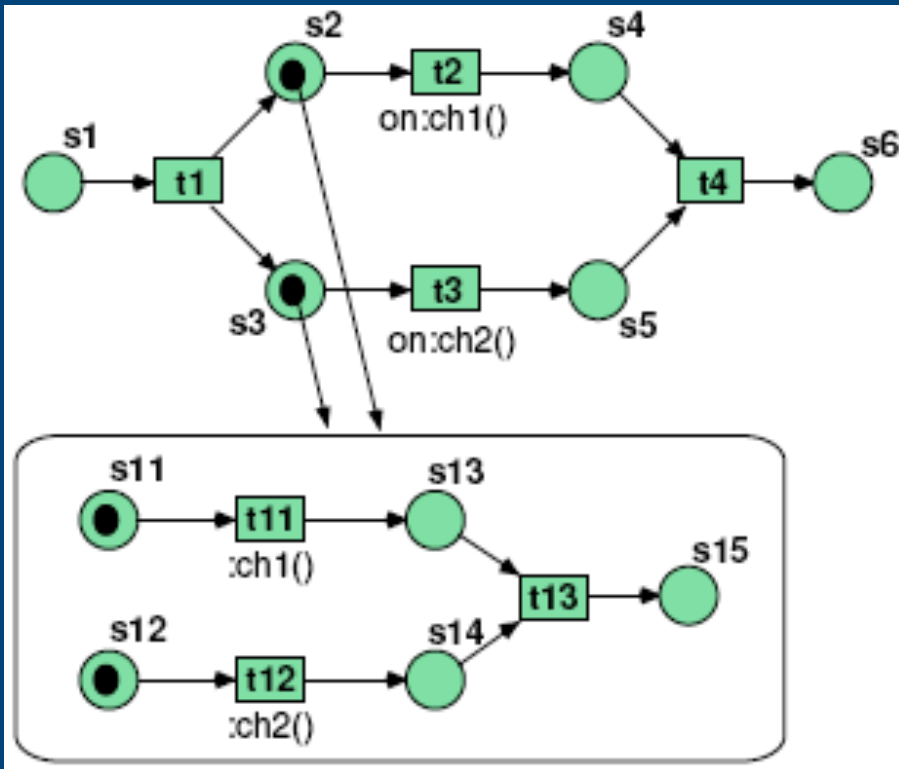


- (At least) two levels of nets:
- **System net** has tokens which are black tokens or object nets
- **Object nets** have black tokens
- **Reference semantics** – tokens can be Object net references
- **Value semantics** – tokens can be Object net instances
- **History process semantics** – tokens can be Object net processes

# Nets-within-nets – reference semantics

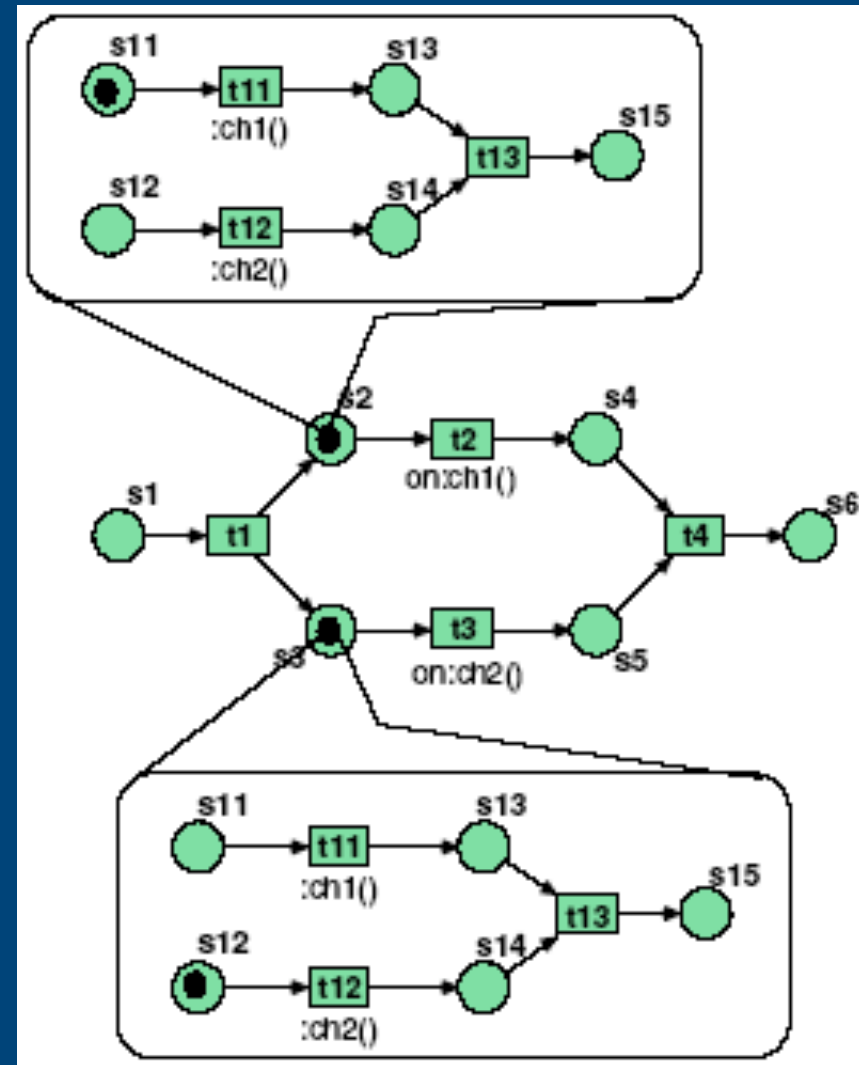
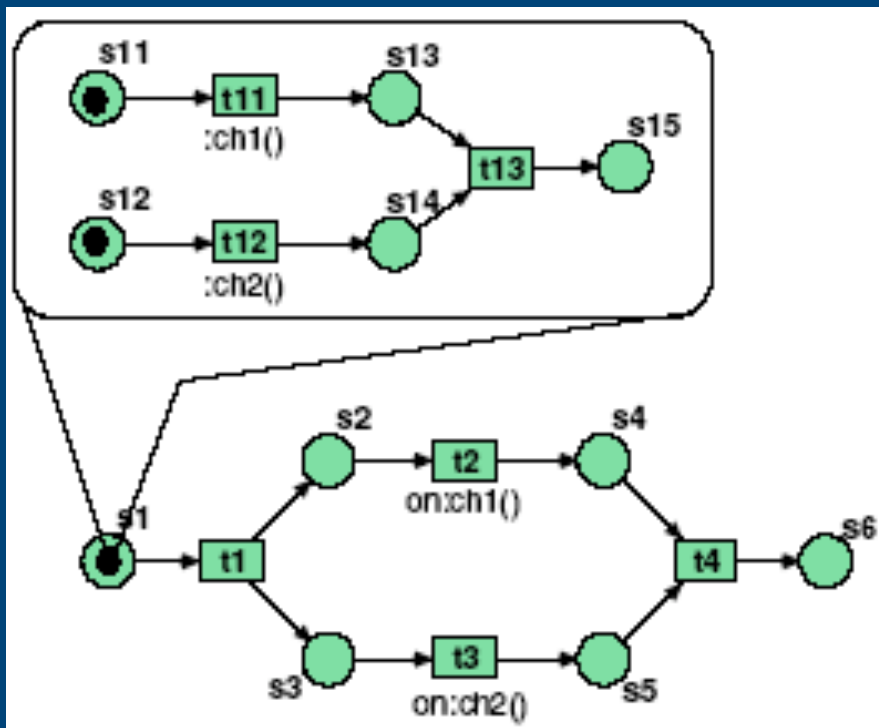


# Nets-within-nets – reference semantics

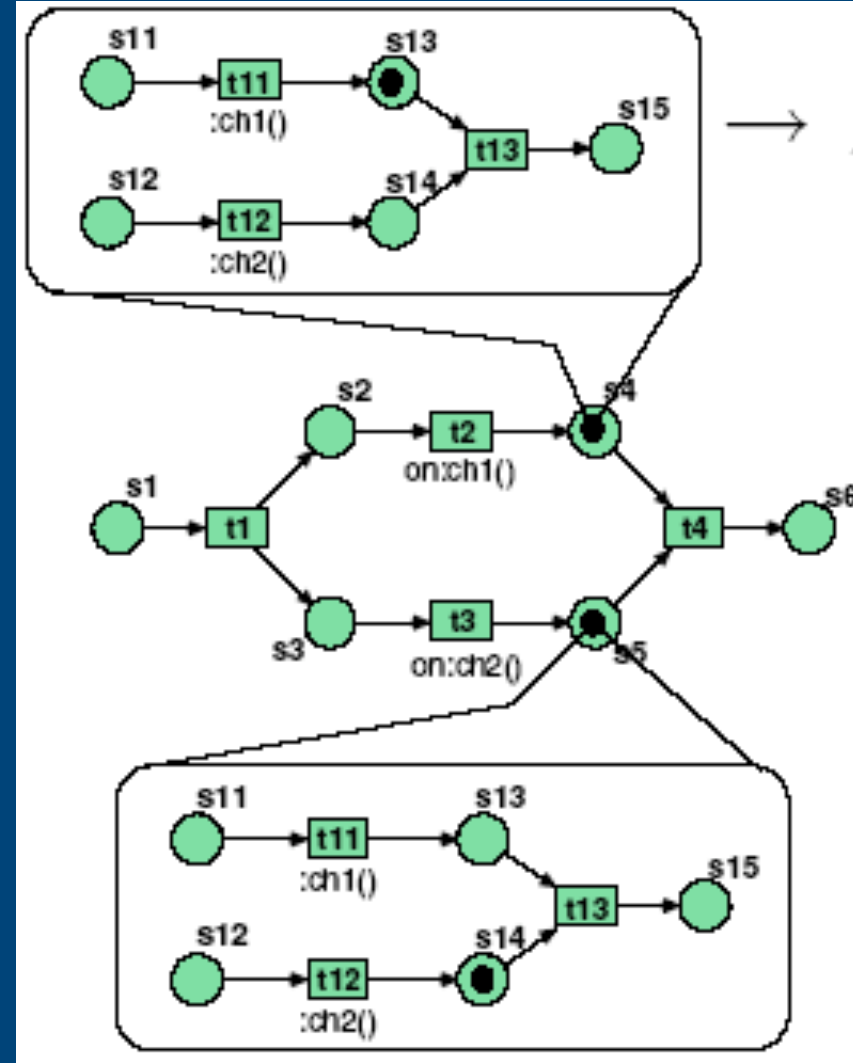
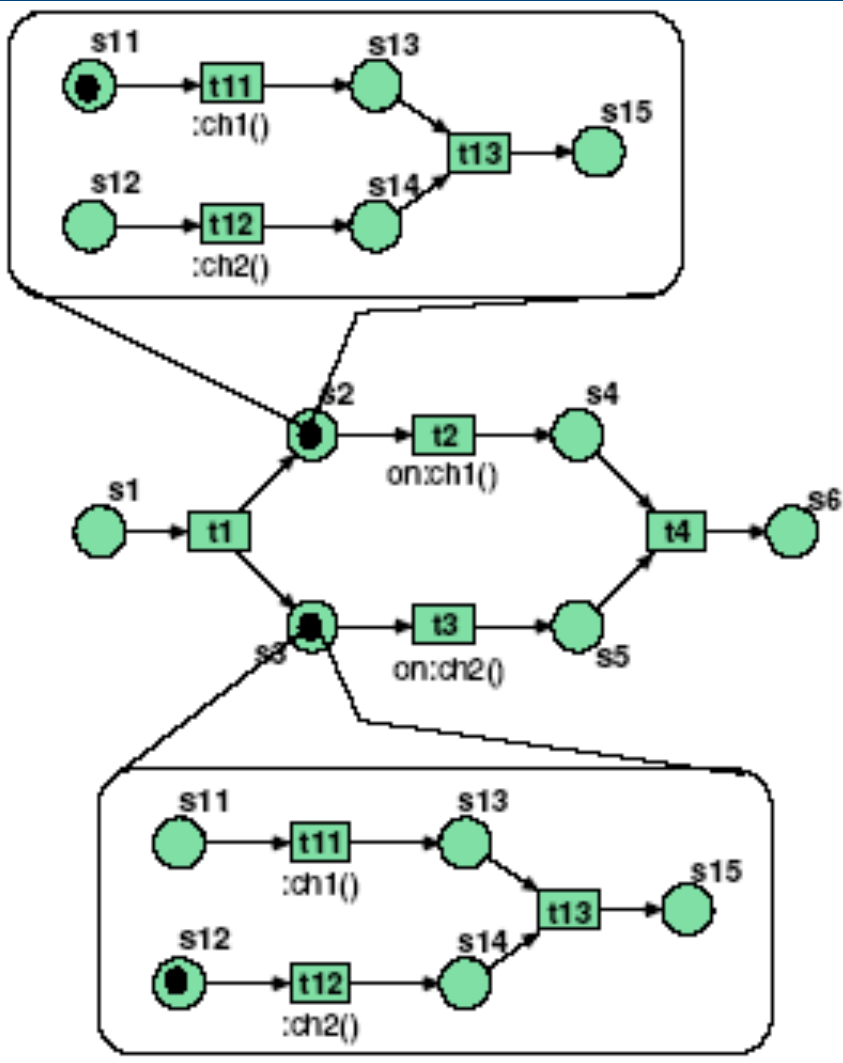




# Nets-within-nets – value semantics



# Nets-within-nets – value semantics



# Nets-within-nets Limitations



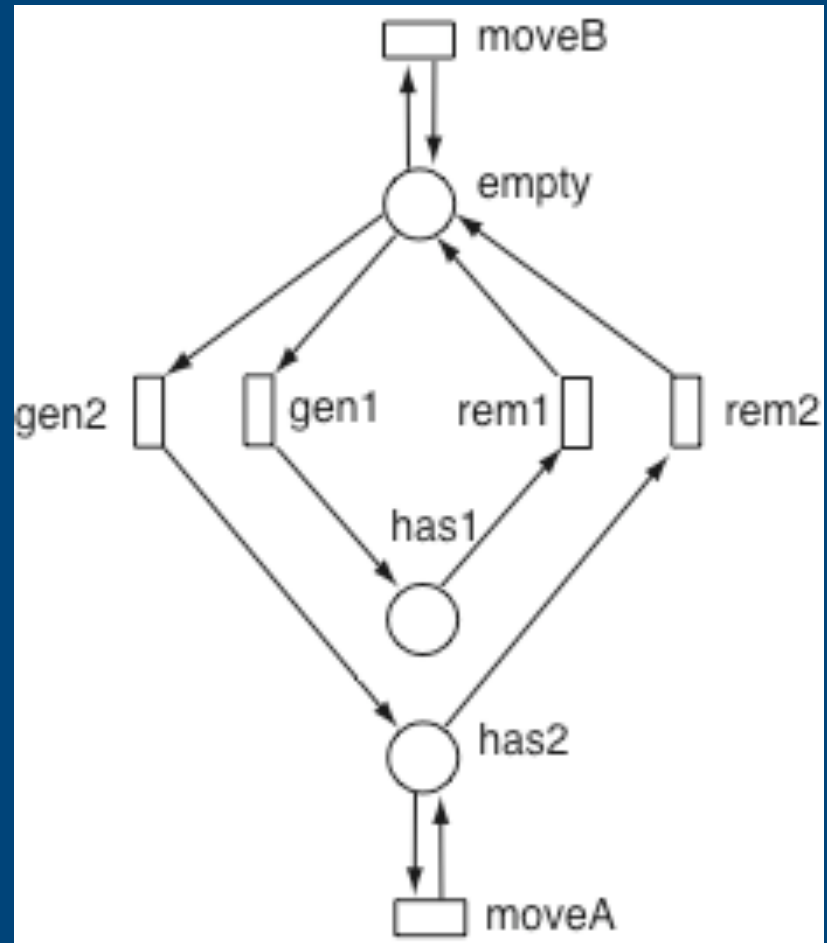
- Either *value* or *reference* or ... semantics
  - Value semantics gives notion of locality
  - Reference semantics gives notion of connectivity
- Limited interaction
  - object net can only interact with transitions adjacent to place
- Formal results are for very limited examples
  - One system net and one (instance of an) object net
  - Value semantics is more powerful than reference semantics
- Examples with *Renew* are not very persuasive

# Proposal for mobile nets

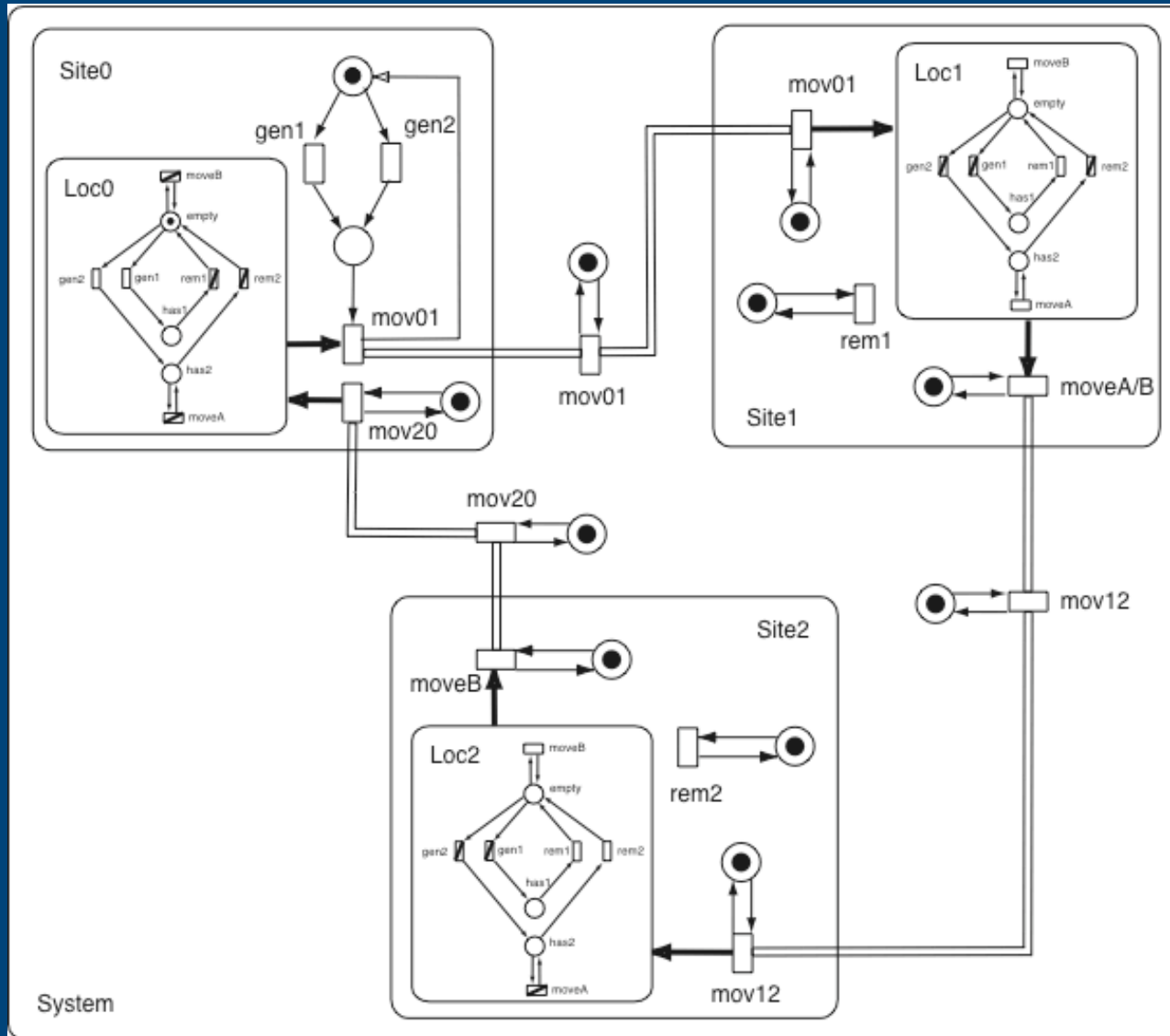


- Start with *modular nets*
  - have a number of Petri Nets – called *modules* or *subnets*
  - combined by place and transition fusion
- Extend the distinction between a *net* and a *system* ...
  - *Subnet* captures the structure of a module
  - *Location* = subnet + fusion context
  - *Subsystem* = location with a non-empty marking

# Mail agent – a subnet



# Mail system



Subnets  
Locations  
Subsystems

Fusions  
Shifting locations

# Nets and locations



## ■ Nets (and subnets) are standard

**Definition 2 (Petri Net).** A Petri Net (PN) is a tuple  $PN = (P, T, W)$  where:

1.  $P$  is a finite set of places.
2.  $T$  is a finite set of transitions with  $P \cap T = \emptyset$ .
3.  $W : (P \times T) \cup (T \times P) \rightarrow \mathbb{N}$  is an arc weight function.

## ■ Locations can be nested (and have a fusion context)

**Definition 5 (PN Location).** A Petri Net Location is a tuple  $L = (S_L, P_L, T_L, W_L)$  where:

1.  $S_L$  is a finite set of locations. We define  $loc(L) = \bigcup_{s \in S_L} loc(s) \cup \{L\}$ . We require  $\forall s \in S_L : loc(s) \cap \{L\} = \emptyset$ .
2.  $(P_L, T_L, W_L)$  is a Petri Net. We define  $plc(L) = \bigcup_{s \in S_L} plc(s) \cup \{P_L\}$  and  $trn(L) = \bigcup_{s \in S_L} trn(s) \cup \{T_L\}$ .

# Mobile systems



- Convenient to specify fusion at the level of the system
  - for convenience we assume transitive closure of place fusion sets
  - for convenience we require consistency of transition fusion sets

**Definition 6 (Mobile System).** A Mobile System is a tuple  $MS = (L_0, PF, TF, M_0)$  where:

1.  $L_0$  is a location, called the root location. We define  $P = plc(L_0)$  and  $T = trn(L_0)$ .
2.  $PF$  is a set of place fusion sets where  $\bigcup_{pf \in PF} pf = P$  and  $\forall pf_1, pf_2 \in PF : pf_1 \cap pf_2 \neq \emptyset \Rightarrow pf_1 = pf_2$ .
3.  $TF$  is a set of transition fusion sets where  $\bigcup_{tf \in TF} tf = T$  and  $\forall tf_1, tf_2 \in TF : tf_1 \cap tf_2 \neq \emptyset \Rightarrow |tf_1| = |tf_2|$ .
4.  $M_0$  is the initial marking of the location.



# Classify places and transitions



- **Local vs exported** – determined by size of fusion sets
- **Vacate vs occupy vs regular** – determined by arcs incident on local places

**Definition 9.** For a Mobile System  $MS$  we classify places and transitions as follows:

1.  $LP = \{p \in P \mid \exists pf \in PF : pf = \{p\}\}$  is the set of local places.
2.  $EP = P - LP$  is the set of exported places.
3.  $LT = \{t \in T \mid \exists tf \in TF : tf = \{t\}\}$  is the set of local transitions.
4.  $ET = T - LT$  is the set of exported transitions.
5.  $VT = \{t \in T \mid \exists p \in LP : W(p, t) > 0 \wedge \forall p \in P : W(t, p) = 0\}$  is the set of vacate transitions.
6.  $OT = \{t \in T \mid \exists p \in LP : W(t, p) > 0 \wedge \forall p \in P : W(p, t) = 0\}$  is the set of occupy transitions.
7.  $RT = \{t \in T \mid \exists p_1, p_2 \in LP : W(t, p_1) > 0 \wedge W(p_2, t) > 0\}$  is the set of regular transitions.

# Well-formed mobile system



- Need to know whether locations are occupied
- Classification of transitions as *vacate*, *occupy*, *regular* is consistent and covers all transitions

**Definition 10 (Well-formed).** A Mobile System  $MS$  is well-formed if:

1. All transitions are vacate, occupy or regular transitions, i.e.  $T = VT \cup OT \cup RT$ .
2. Vacate transitions empty a location for all reachable markings, i.e.  $\forall L \in loc(L_0) : \forall t \in VT \cap T_L : \forall M \in [M_0] : M[t]M' \Rightarrow \forall p \in LP \cap plc(L) : M'(p) = \emptyset$ .
3. Occupy transitions fill a location for all reachable markings, i.e.  $\forall L \in loc(L_0) : \forall t \in OT \cap T_L : \forall M \in [M_0] : M[t]M' \Rightarrow \forall p \in LP \cap plc(L) : M(p) = \emptyset$ .

# Isolated subsystem



- An *isolated subsystem* has no effect (directly or indirectly) on the root location
  - it can be ignored for the purposes of reachability analysis

**Definition 11 (Isolated subsystem).** *Given a Mobile System  $MS$  in marking  $M$ , a transition sequence  $t_1 t_2 \dots t_n$  is a causal sequence if there are markings  $M_1, M_2, \dots, M_n$  such that  $M[t_1 \rangle M_1[t_2 \rangle M_2 \dots [t_n \rangle M_n$  and  $\forall k \in 1..(n-1) : \exists p \in P : W(t_k, p) > 0 \wedge W(p, t_{k+1}) > 0$ . Given a Mobile System  $MS$ , a subsystem resident in location  $L$  is isolated in marking  $M$  if there is no causal sequence  $t_1 t_2 \dots t_n$  with  $t_1 \in T_L$  and  $t_n \in T_{L_0}$ .*

# Coloured mobile systems



- Adopt the common approach of using colour to distinguish folded components – see def 20
- Require such colours to be used **consistently**
  - identifiers determine the associated subsystems
  - distinct subsystems have distinct identifiers
  - tokens in fused places indicate **all** subsystems to which it belongs
  - firing modes of fused transitions indicate **all** participating subsystems
  - transition firing modes must have identifier in common with tokens
  - transitions cannot **invent** identifiers matching existing subsystems

# Coloured mobile systems



- **Colour makes things more concise but more messy formally**
  - there are many choices for coding token and mode colours
  - the imposed regularity helps in determining properties
- **Isolated subsystems can be defined using causal sequences**
- **Can be approximated with garbage collection algorithms**
  - for a location to modify the root location, you need fusion
  - fusion requires that the context knows the identifier of the subsystem
  - this condition is sufficient to imply that the subsystem is isolated but it is not a necessary condition

# Conclusions



- **A natural way to capture mobility in the Petri Net formalism**
  - start with modular nets (with general fusion possibilities)
  - differentiate subnets, locations, subsystems
- **Well-formed property is based on classifying transitions**
  - tells us when a subsystem migrates from one location to another
- **Isolated subsystems (garbage) cannot affect the root location**
  - notion is difficult to compute precisely – colour helps to approximate
- **State space exploration is possible using symmetry techniques**
  - see ATVA 2005 paper

# State Space Exploration of Object-Based Systems using Equivalence Reduction and the Sweepline Method

(ATVA 2005)

Charles Lakos

# Roadmap



- Characteristics of Object-Based Systems
- State space exploration requirements
- Equivalence reduction + Sweepline
- Experimental results
- Conclusions



# Object-Based Systems



- Object-Based Systems have the notion of **objects**
- Object-Oriented Systems also include the notion of **inheritance**
- “An **object** has **state**, **behaviour** and **identity**”
  - **state** = static properties together with their current values
  - **behaviour** = how an object acts and reacts with changes of state
  - **identity** = property that distinguishes one object from all others

# Object Identity



- Object identity is a key feature of object systems
  - it implies some form of reference semantics
- The analysis of object-based systems will require techniques to handle object identity
  - specific object identifiers are not important but only equality
  - allocation of objects in a concurrent/distributed system will result in objects with different identifiers but the same essential configuration
- We need some form of equivalence reduction or graph isomorphism

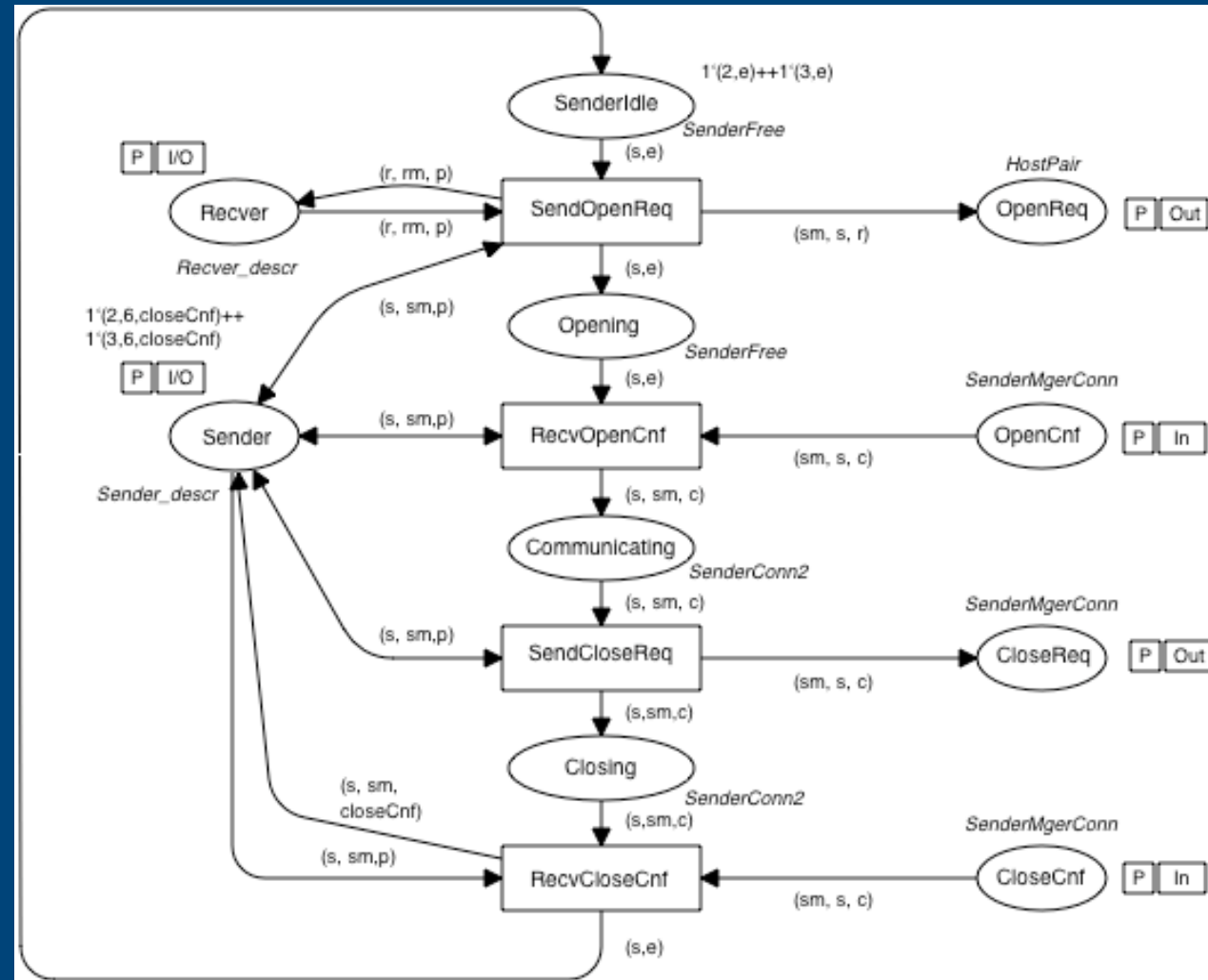
# Range of applicability



- Clearly relevant to the analysis of object-oriented software
  - important given the wide adoption of OO technology
- Also relevant to mobile and agent-oriented systems
  - a device or process migrates and changes locality while retaining its connectivity (and its identity)
  - an agent has self-contained functionality and migrates to achieve efficiency gains while retaining references to its initiator and/or target

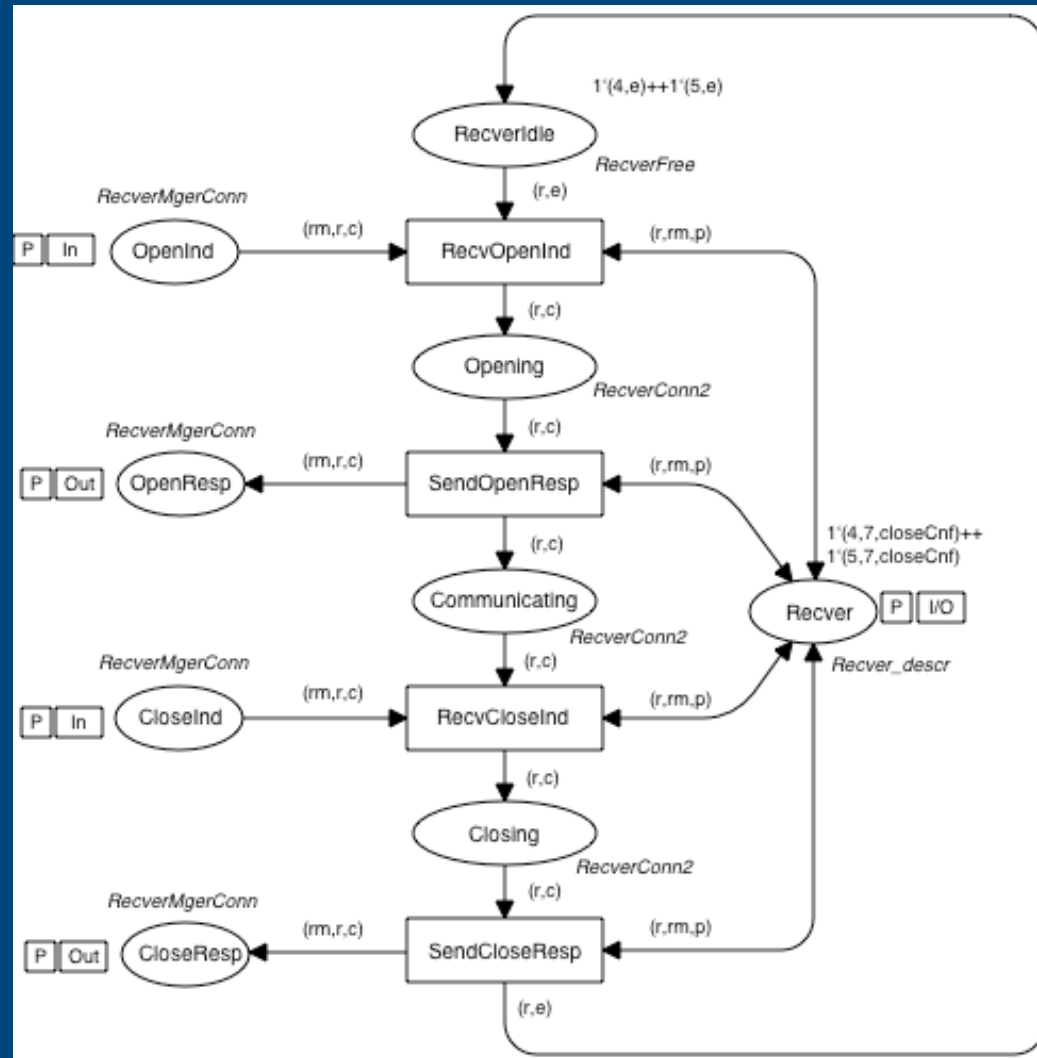
# Example: Protocol for confirmed establishment of connections

- Sender



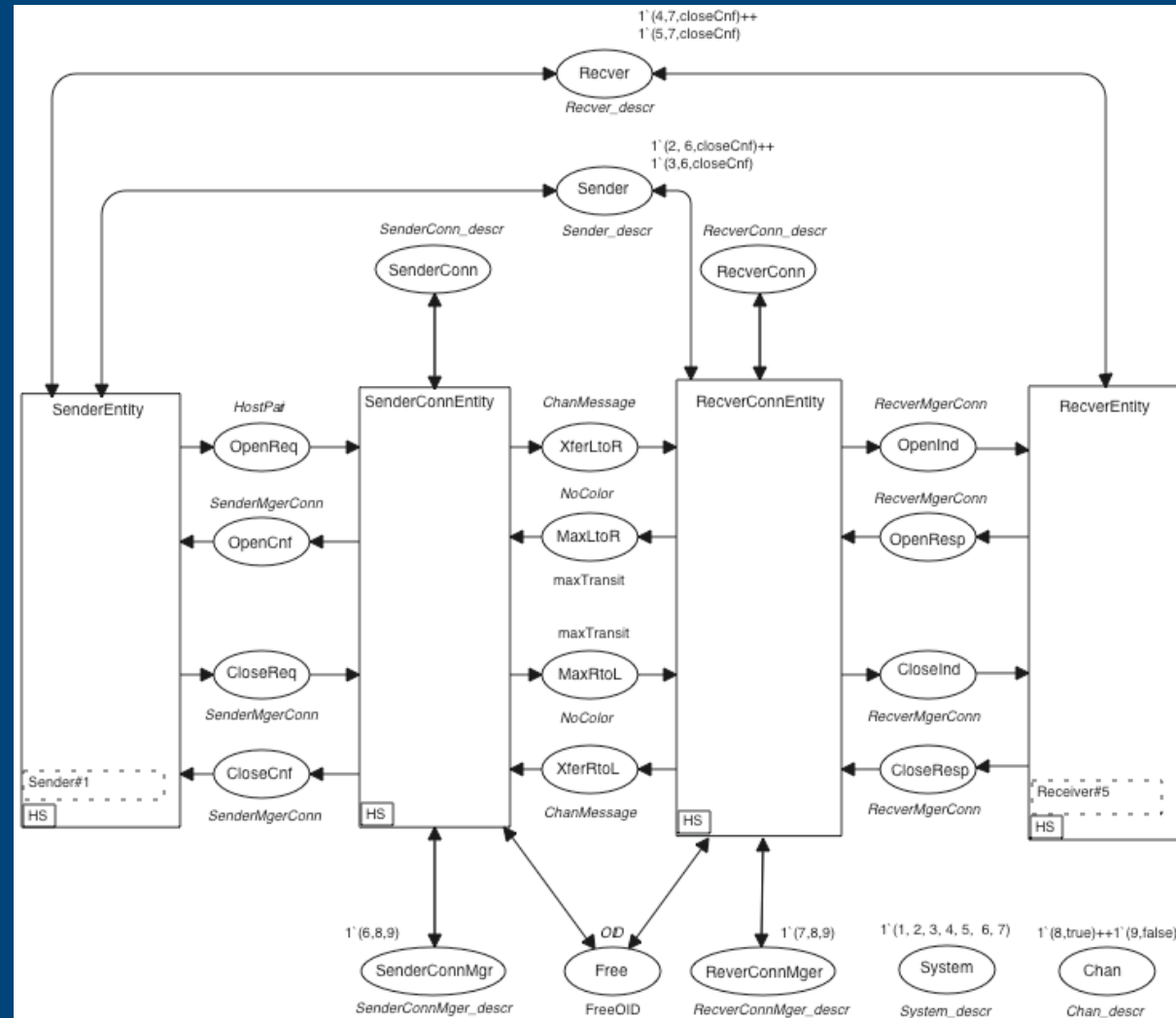
# Example: Protocol for confirmed establishment of connections

- Receiver



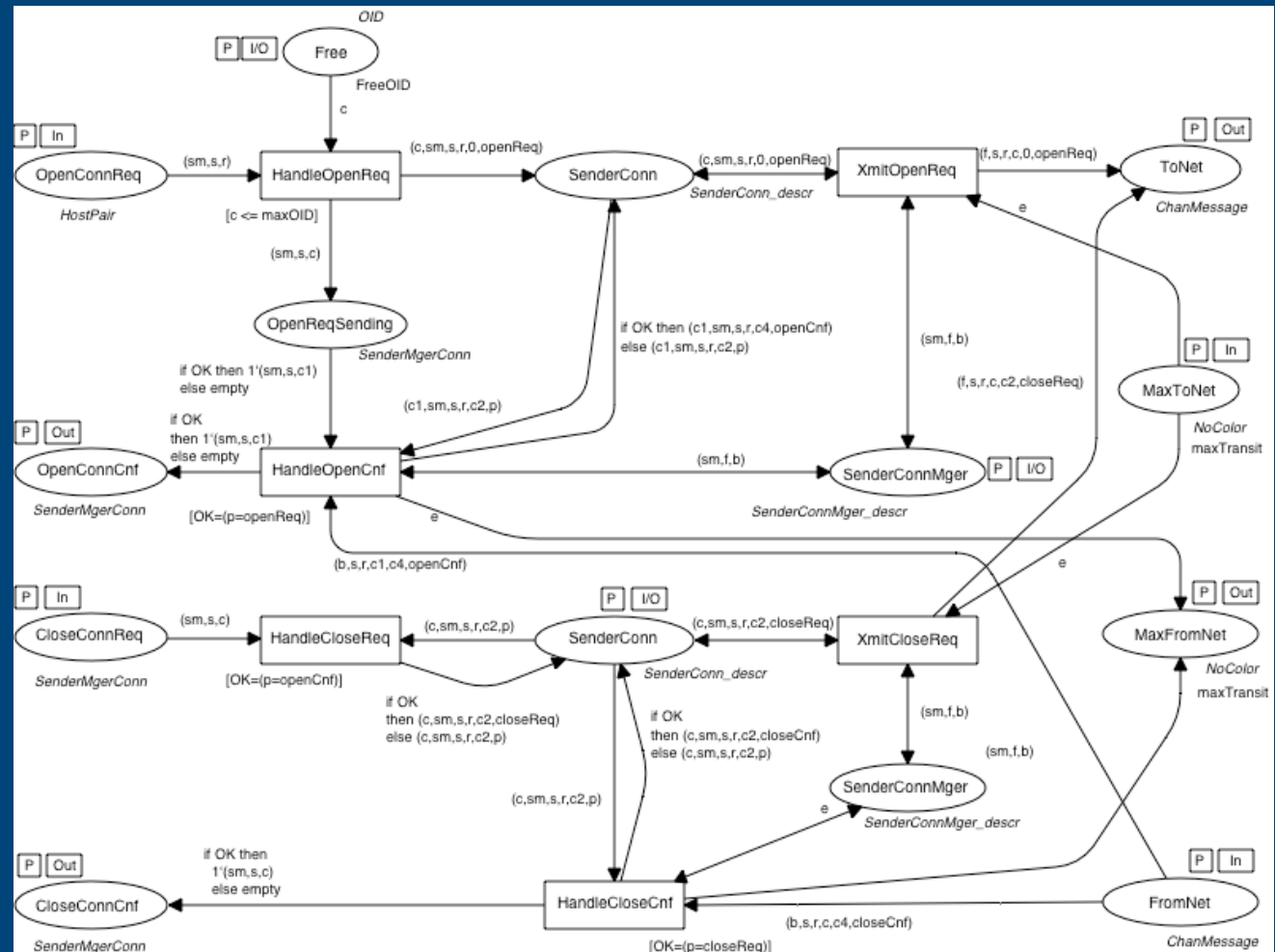
# Example: Protocol for confirmed establishment of connections

- System



# Example: Protocol for confirmed establishment of connections

- Sender manager



# Notes on the protocol example



- All components are objects
  - senders, receivers, connection managers, connections
  - only connections are dynamically allocated and discarded
- The *behaviour* of an object is given by a Petri net
- The *state* of an object is given by its marking
- The *identity* of an object is given by an integer object identifier
  
- Multiple instances are folded onto the one subnet with object identifiers to distinguish the instances



# State space exploration for object-based systems



- Basis of model-checking
- Primary obstacle is *state space explosion*
- Need to adopt equivalence reduction so that states which are essentially the same are treated as such
- Need to eliminate garbage which could otherwise unnecessarily differentiate states

# Raw state space results



Free = number of available identifiers  
– each end of a connection requires one identifier

Identifiers allocated in any order

Identifiers allocated in sequence

Free	Full		
	Nodes	Arcs	Sec
3	9,897	28,716	9
4	256,617	826,540	1,794
3	3,153	8,064	2
4	48,725	155,680	89
5	87,029	284,272	403
6	>251,500		>1 hr

# The dSPIN approach



- A depth-first traversal is performed of the system state
- Object identifiers are reallocated in the order of traversal
  - this produces a (unique) canonical representation
- The depth-first traversal is also used for garbage collection
  - a mark-and-sweep algorithm
- This approach does not deal with unordered collections of references
  - a general question of symmetry or graph isomorphism

# Raw results and dSPIN algorithm (without enhancement)

Free	Full			<i>Canon</i> <sub>1</sub>		
	Nodes	Arcs	Sec	Nodes	Arcs	Sec
3	9,897	28,716	9	1,538	4,532	1
4	256,617	826,540	1,794	7,524	24,896	7
5				8,982	32,806	10
6				10,712	42,820	13
7				10,720	48,268	14
16				10,720	96,940	24
3	3,153	8,064	2	1,538	4,480	1
4	48,725	155,680	89	7,524	24,096	7
5	87,029	284,272	403	8,982	30,786	9
6	>251,500		>1 hr	10,712	36,736	12
7				10,720	37,592	12
16				10,720	37,592	12

# Sweeping method for reducing memory demands

- Applicable to systems which exhibit a notion of progress
  - states with an earlier progress value cannot be revisited from states with a later progress value
  - states with earlier progress values can be discarded
- The method can be extended to cater for *regress* edges
- E.g. protocol with numbered messages
- E.g. timed systems
- E.g. object-based systems

# Experiments with three progress measures



- $\psi_1$  = next available (numeric) object identifier
- $\psi_2$  = weighted sum of connection progress (4 steps)
- $\psi_3$  = weighted sum of senders (16 steps)

Free	Full			Sweep-Line $\psi_1$		Sweep-Line $\psi_2$		Sweep-Line $\psi_3$	
	Nodes	Arcs	Sec	Peak	Sec	Peak	Sec	Peak	Sec
3	9,897	28,716	9	8,088	12	2,976	11	2,304	13
4	256,617	826,540	1,794	226,320	1,925	54,528	1,767	61,152	3,145
3	3,153	8,064	2	2,352	2	720	2	768	2
4	48,725	155,680	89	45,572	100	10,752	112	12,112	123
5	87,029	284,272	403	60,788	543	16,576	557	15,840	459
6	>251,500		>1 hr						

# Combining equivalence reduction and sweepline



- Superficially contradictory
  - equivalence reduction looks to match already visited states
  - sweepline aims to avoid reconsidering prior states
- But examples commonly mix the two:
  - protocols with numbered messages may use cyclic numbering
  - timed systems may exhibit repeated patterns of behaviour
  - object-based systems may exhibit cyclic behaviour as objects are allocated and discarded

# Combining equivalence reduction and sweepline



Free	Full			$Canon_1$			Combined $Canon_1, \psi_2$				Combined $Canon_1, \psi_3$			
	Nodes	Arcs	Sec	Nodes	Arcs	Sec	Nodes	Arcs	Peak	Sec	Nodes	Arcs	Peak	Sec
3	9,897	28,716	9	1,538	4,532	1	3,074	9,056	445	3	2,774	8,124	301	1
4	256,617	826,540	1,794	7,524	24,896	7	15,046	49,784	2,223	17	18,613	61,304	1,957	23
5				8,982	32,806	10	17,962	65,604	2,999	23	20,515	75,474	2,189	28
6				10,712	42,820	13	21,420	85,624	4,171	30	24,107	97,320	2,603	35
7				10,720	48,268	14	21,436	96,518	4,179	31	24,139	108,966	2,611	38
16				10,720	96,940	24	21,436	193,844	4,179	55	24,283	213,060	2,611	66
3	3,153	8,064	2	1,538	4,480	1	3,074	8,952	445	3	2,774	8,020	301	2
4	48,725	155,680	89	7,524	24,096	7	15,046	48,184	2,223	16	18,613	59,048	1,957	21
5	87,029	284,272	403	8,982	30,786	9	17,962	61,564	2,999	21	20,507	69,850	2,189	25
6	>251,500		>1 hr	10,712	36,736	12	21,420	73,458	4,171	25	24,091	82,222	2,603	30
7				10,720	37,592	12	21,436	75,170	4,179	27	24,107	83,562	2,611	31
16				10,720	37,592	12	21,436	75,170	4,169	27	24,107	83,562	2,611	31



# Relating canonicalisation and progress



## ■ Progress measures:

- $\psi_1$  = next available (numeric) object identifier
- $\psi_2$  = weighted sum of connection progress (4 steps)
- $\psi_3$  = weighted sum of senders (16 steps)

## ■ Canonicalisation functions:

- $\text{Canon}_1$  = depth first traversal taking natural order of tokens in places
- $\text{Canon}_2$  = order senders by progress measure  $\psi_3$

# Canonicalisation results



Free	Full			<i>Canon<sub>1</sub></i>			<i>Canon<sub>2</sub></i>		
	Nodes	Arcs	Sec	Nodes	Arcs	Sec	Nodes	Arcs	Sec
3	9,897	28,716	9	1,538	4,532	1	421	1,230	1
4	256,617	826,540	1,794	7,524	24,896	7	2,053	6,755	1
5				8,982	32,806	10	2,467	8,953	2
6				10,712	42,820	13	2,981	11,803	3
7				10,720	48,268	14	2,989	13,368	3
16				10,720	96,940	24	2,989	27,093	6
3	3,153	8,064	2	1,538	4,480	1	421	1,213	1
4	48,725	155,680	89	7,524	24,096	7	2,053	6,523	1
5	87,029	284,272	403	8,982	30,786	9	2,469	8,378	2
6	>251,500		>1 hr	10,712	36,736	12	2,981	10,116	3
7				10,720	37,592	12	2,989	10,408	3
16				10,720	37,592	12	2,989	10,408	3

# Combined results for Canon<sub>2</sub>



Free	Full			<i>Canon</i> <sub>2</sub>			Combined <i>Canon</i> <sub>2</sub> , $\psi_2$				Combined <i>Canon</i> <sub>2</sub> , $\psi_3$			
	Nodes	Arcs	Sec	Nodes	Arcs	Sec	Nodes	Arcs	Peak	Sec	Nodes	Arcs	Peak	Sec
3	9,897	28,716	9	421	1,230	1	840	2,452	125	1	753	2,175	89	1
4	256,617	826,540	1,794	2,053	6,755	1	4,101	13,485	598	4	5,092	16,660	493	5
5				2,467	8,953	2	4,934	17,887	808	5	5,673	20,683	551	7
6				2,981	11,803	3	5,955	23,560	1,199	7	6,741	26,898	735	9
7				2,989	13,368	3	5,972	26,692	1,207	8	6,759	30,234	739	10
16				2,989	27,093	6	5,972	54,124	1,207	14	6,798	59,621	738	17
3	3,153	8,064	2	421	1,213	1	840	2,418	125	1	753	2,141	89	1
4	48,725	155,680	89	2,053	6,523	1	4,102	13,025	598	4	5,092	16,024	493	5
5	87,029	284,272	403	2,469	8,378	2	4,933	16,733	808	5	5,674	19,125	551	6
6	>251,500		>1 hr	2,981	10,116	3	5,956	20,198	1,199	6	6,741	22,763	735	8
7				2,989	10,408	3	5,971	20,778	1,207	7	6,756	23,250	739	8
16				2,989	10,408	3	5,971	20,778	1,207	7	6,756	23,250	739	8

# Conclusions



- It is important to identify a good canonicalisation function for the state space exploration of object-based systems
  - in general, this is a difficult problem
- The sweepline method identifies a notion of progress
  - this can be used to conserve memory during state space exploration
  - it can also be used to define a canonicalisation function
- Results indicate that the same progress measure can be used for both purposes